

Space-Time Evolution of the Oscillator, Rapidly Moving in a Random Media

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Abstract

We study the quantum-mechanical evolution of the nonrelativistic oscillator, rapidly moving in the media with the random vector fields. We calculate the evolution of the level probability distribution as a function of time, and obtain rapid level diffusion over the energy levels. Our results imply a new mechanism of charmonium dissociation in QCD media.

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I. INTRODUCTION.

The propagation of the charged particles, both quantum and classical, through different types of media had long ago become an important branch of theoretical physics. (See e.g. an excellent review [1] for the discussion on the present status of the subject). However, much less is known about the propagation of neutral bound systems and the wave packets with the zero total charge but nonzero higher multipole moments. This is especially true for quantum systems. Recently there was a revived interest in the subject. This is due to numerous possible applications in nuclear and particle physics, especially in the study of exclusive processes and the creation and the diagnostics of the quark-gluon plasma [2]. Such systems can be also studied in electrodynamics, where it is possible to observe the propagation of e^+e^- positronium through the random media.

Although a significant advance in the study of the propagation of these objects was made recently, very little is known about the evolution of the fast wave the packets and the dipoles in the random media, and it's influence on the particle and nuclear cross-sections, diffractive processes and energy losses.

The aim of this paper is to study the propagation of the highly energetic, but nonrelativistic (in it's c.m. rest reference frame) electrically neutral dipoles and wave packets, with nonzero dipole moment, and with the internal interactions, through the random media. For simplicity, and in order to have an almost explicit solution we shall consider the simplest example of such kind – a harmonic oscillator moving through the media with the random vector fields. The latter can be any media, that where there are random vector fields, i.e. the fields transforming as vectors under the Lorentz group transformations. In nature these are electromagnetic and Yang-Mills fields. We shall call below any such media a vector media. The examples of the vector random media include amorphous solid state media [3], usual electromagnetic plasma [4], dense QCD media [5], and quark-gluon plasma [6,7]. It is less clear what happens in the nuclear media or nuclei, where the fields are colorless and it is not obvious how they transform under Lorentz transformations.

In our paper we consider the nonrelativistic oscillator, whose center of mass moves with the relativistic speed through the random media, which is characterized by the static random electric fields. In our simple model, the particles inside the dipole interact via harmonic oscillator potential (no Coulomb interactions between them), while the interaction with the external field is electromagnetic. However we shall argue below that at least qualitatively our results also hold for the interaction with the QCD colored fields, and for the general law of interaction between the constituents of the dipole.

Our main result will be the development of the general formalism that will allow us to determine the density matrix, expansion rate and level diffusion for nonrelativistic harmonic oscillator moving with high velocity in the random media. We shall derive the effective action and the effective transition operator for the bound system propagation in the random media. This effective action will be used to study two closely connected problems: the propagation of the small initial size dipole/wave packet through the random media and of the oscillator in the ground state through the random media. We shall determine the energy level diffusion rate of the small wave packet for small and big times, as well as the probability that it occupies level n after time t . We shall also determine the expansion rate of the ground state and the probabilities that after time t the oscillator will stay in the ground state or will

move into the state with the energy level N .

Taking into account that the electric fields transform as the components of the rank 2 tensor during the Lorentz transformation, while the time as the component of the 4-vector we shall obtain for a certain interval of times the superdiffusion expansion law for the wave packet density radius:

$$\rho^2 = (A_0 t^2 + A_1 t^3)/\gamma^2 \quad (1.1)$$

Here $A_0 \sim d - 1/d$ describes the quantum mechanical expansion rate of the wave packet and A_1 is the coefficient depending on the properties of the media, γ is the Lorentz factor and ρ is the radius of the dipole, and d is the inverse squared radius of the initial state (for the ground state $d=1$). Thus, random media may strongly influence the cross-sections and energy losses. The physical reason for "superdiffusion" is the extraction of the resonant (i.e. with oscillator proper frequency) mode from the spectrum of the random electric field fluctuations.

This superdiffusion effect is very similar to the one discussed recently in refs. [8,9] in the context of the random walk of the classical particles in the time dependent random potential in statistical mechanics.

Our results imply a new mechanism for charmonium dissociation in vector media: it is excited to higher energy levels due to the scattering on random electric fields, and then dissociates into $D\bar{D}$ pairs [10,11].

The calculation will be made in the eikonal approximation, i.e. we shall assume that the center of mass of the dipole moves along the straight line with the ultrarelativistic constant velocity $v_0 \approx c$, where c is the speed of light. We shall neglect in this approximation both the deviations of the center of mass from the straight line, and the velocity changes due to the interaction with the random fields. The latter corrections will be taken into account elsewhere. We shall assume that the size of our dipole is small relative to the inhomogeneity scale of the media $\approx 1/\kappa$.

As it was mentioned above, all our calculations will be carried in the simplified model of the dipole with the oscillator interaction between the constituents and electromagnetic interaction with the external field. We neglect the Coulomb interaction between internal constituents of the dipole. We expect that our result will not change qualitatively if we generalize the external field to a color one, and take into account Coulomb interaction between the constituents.

We shall see that the influence of the random media on the oscillator has very transparent mechanical analogue in the classical problem of the excitation of the oscillator by the external resonance force [12]. The random media obviously has the role of this external force. On the other hand previous investigations, using this analogy, correspond to the study of the proper oscillations of the oscillator.

Some of the aspects of the problem of the dipole propagation were considered before [2], [13], [14], but the latter authors did not take into account the influence of the media on the propagation of the dipole and its expansion, and the change of its quantum state during the propagation. They only considered the interaction free dipole, neglecting the influence of the media on the dipole structure and the internal interactions. Although as we shall see the internal interactions do not lead to new qualitative effects (as it was noted in ref.

[13]), the interaction with the media does play a major role if time is not too small. The only previous attempt to take into account the influence of the media on the propagation of the oscillator in the media was made in ref. [2,14], but in the different context. The latter authors considered not vector, but scalar media, in particular nuclei, and parameterized it's influence by taking into account absorption.

The problem of harmonic oscillator propagation naturally arises in the context of the propagation of the bound systems like charmonium through the quark-gluon plasma.

Note that there are strictly speaking two different problems that can be discussed in the connection with the oscillator: one is the propagation of the two-particle bound state-dipole through the media, another is the propagation of the wave packet. In the approximation used in this paper we shall see that technically this is the same problem. The quantum dynamics of the bound state can be reduced to the quantum dynamics of the wave packet. So below, if not stated otherwise we shall not distinguish between them. However, beyond the approximation of the homogeneous electric field discussed below, these are two different problems, with different Lagrangians.

The paper is organized as follows. In the second chapter we shall develop the general formalism for the description of the quantum dipole moving through the random media. First, we shall derive the effective Lagrangian for the propagating oscillator in this media, which we shall model by the usual plasma. We shall neglect the radiation. The discussion of this important phenomena will be considered in a separate paper. Next we shall consider the simplified model, that leads to the same qualitative results. We shall show that in the eikonal approximation it is sufficient to consider the behavior of the oscillator in the time-dependent homogeneous electric field. We shall calculate using the functional integral method the effective evolution operator and the probability to find an oscillator at particular levels as a function of time. In the third chapter we shall find the averaged density matrix and study the expansion of the small wave packets and see the influence of the media on the wave packet expansion. In the fourth chapter we shall consider the probability evolution for the ground state wave packets and apply our results to the simplest model of charmonium. We summarize our main results in conclusion. Some details of the calculations will be discussed in Appendices.

II. THE MODEL OF THE OSCILLATOR EVOLUTION.

A. The Formulation of the Problem.

The most widespread model of the random media where the bound system can propagate is the conventional model of plasma [4]. Such model can be used qualitatively to describe the amorphous material. The media is represented by a random set of ions that generate electric potentials with the Debye screening.

$$V(r) = g \exp(-\kappa r)/r \quad (2.1)$$

Here g is the electric charge. If we neglect in this model three particle correlations, the potential-potential correlation function in such system is ([4])

$$L(\vec{r} - \vec{r}') = \langle V(\vec{r})V(\vec{r}') \rangle = A^2 \exp(-\kappa|\vec{r} - \vec{r}'|)/|\vec{r} - \vec{r}'| \quad (2.2)$$

A is the constant dependent on the properties of the media. We will assume that the correlations are gaussian, i.e. that all correlation functions can be expressed through the two-point correlation function (2.2). This is the usual assumption in the statistical mechanics. More complicated systems with nongaussian correlations are usually highly nontractable.

For the QCD media or the quark-gluon plasma the most wide-spread model is the model discussed in ref. [15]. This model is very similar to the conventional model of plasma, discussed above, except that instead of the conventional electric charges the particles have the color charges T^a , that are the generators of the $SU(3)$ color group, while the interactions contain factors $g^2 T^a T^b$ that are very similar to the charge products. If we take into account only the binary interactions, the plasma becomes color-diagonal, and we can neglect the color factors, that will not make qualitative influence on our results. The potentials and the correlation functions are the same as in eq. (2.2) (except some numerical factors whose influence will be discussed in a separate publication).

Note that in terms of the language of the statistical mechanics for all cases (plasma, QCD media, quark-gluon plasma) our problem is the well known problem of the description of the propagation of the wave packet in the random gaussian media characterized by given correlation functions (see Appendix A for a more detailed discussion). Consequently we consider in this paper all types of plasma as an electromagnetic plasma.

Consider now the fast moving neutral system of two particles (dipole) with a harmonic interaction between it's components in the center of mass reference frame:

$$W(|\vec{r} - \vec{r}'|) = N|r - r'|^2 \quad (2.3)$$

The system is moving in a random media discussed above with a velocity $v_0 \approx c$, where c is the speed of light. For this ultrarelativistic speed we can use the eikonal approximation. We assume that the center of mass moves along the straight line with a constant velocity. Radiation will be neglected.

According to our assumption the oscillator is nonrelativistic, consequently we can separate the motion of two constituents of the dipole into the motion of the center of mass and the relative motion of the two particles (see ref. [16] and Appendix B for detailed discussion):

$$L = L_{c.m.} + L_{n.r.} \quad (2.4)$$

In the eq. (2.4) the first term describes the free motion of the center of mass with the speed v_0 , while the second is the Lagrangian of the relative nonrelativistic motion of the two components of the dipole in the c.m. reference frame. The latter Lagrangian can be written for the case of the small dipole with the size $\ll 1/\kappa$,

$$L_{n.r.} = \frac{1}{2}M(\dot{u}^2 - \omega_0^2 \vec{u}^2) - g(V(\vec{r}_1, t) - V(\vec{r}_2, t)) \quad (2.5)$$

Here V is the electric potential in the oscillator c.m. frame, \vec{r}_1, \vec{r}_2 are the coordinates of the dipole components in this frame, and $\vec{u} = \vec{r}_1 - \vec{r}_2$ is the relative coordinate of two constituents of the dipole. The derivation of the Lagrangian (2.5) is discussed in Appendix B.

For the small size dipole we may use a Taylor expansion:

$$V(\vec{r}_1, t) - V(\vec{r}_2, t) = (\vec{r}_1 - \vec{r}_2) \frac{\partial V(0, t)}{\partial \vec{r}} \quad (2.6)$$

Here $V(0, t)$ is the electric potential in the c.m. reference frame at the moment t (t here is the laboratory time) that corresponds to the random potential in the point where the center of mass is located, it's derivative is obviously the electric field in this point. Thus for the small size dipole, with the size $\ll 1/\kappa$, we can neglect inhomogeneity of the field and reduce the problem of the dipole motion to the problem of the oscillator in the homogeneous electric field

$$\vec{E} = -\frac{\partial V(0, t)}{\partial \vec{r}} \quad (2.7)$$

(measured in the oscillator c.m. frame in the point corresponding to the oscillator c.m.). In this approximation, and this is an approximation that we will use in the present paper, the problem of the moving oscillator is equivalent to the problem of the propagation of the wave packet in the electric field \vec{E} . For the larger dipole we must use the full second term in the Lagrangian (2.5), taking into account the inhomogeneity of the field.

The same separation can be made for the nonrelativistic neutral wave packet, whose center of mass moves with a relativistic speed. In this case the nonrelativistic Lagrangian that describes the space-time evolution of the dipole in it's rest frame can be written as

$$L_{\text{n.r.}} = \frac{1}{2}M(\dot{\vec{r}}^2 - \omega_0^2 \vec{r}^2) - gV(\vec{r}, t) \quad (2.8)$$

Here \vec{r} is the distance between two particles in the rest frame of the dipole or the radius vector in the c.m. at rest reference frame for a wave packet. V is the potential in the wave packet c.m. reference frame. The Lagrangian (2.5) in the approximation (2.6) is evidently a particular case of eq. (2.8).

Let us go to the rest frame of the dipole. In the latter frame our problem is evidently a problem of the nonrelativistic oscillator with the internal interaction given by eq. (2.3 and in the external electromagnetic field.

Electromagnetic field acting on the particles that built up the oscillator can be easily found by making the Lorentz transformation [17]. (Note that within the present approximation, since the relative motion is nonrelativistic, the relative speeds of the two particles can be neglected to perform the Lorentz transformation according to c.m. speed v_0).

After the Lorentz transformation (see appendix B for details), the laboratory frame 4-potential $(V, 0, 0, 0)$ of the electromagnetic fields becomes $(V_{\text{c.m.}}, 0, 0, A_3)$:

$$V_{\text{c.m.}} = V\gamma, \quad A_3 = -V\gamma$$

The electromagnetic tensor \vec{E}, \vec{B} ($\vec{B} = 0$ in the laboratory reference frame) becomes:

$$\begin{aligned} E_{\text{c.m.}1,2} &= E_{1,2}\gamma, \quad E_{\text{c.m.}3} = E_3 \\ B_{\text{c.m.}1} &= v_0 E_2 \gamma, \quad B_{\text{c.m.}2} = -v_0 E_1 \gamma, \quad B_{\text{c.m.}3} = 0 \end{aligned} \quad (2.9)$$

Here γ is the Lorentz factor:

$$\gamma = 1/\sqrt{1 - v_0^2/c^2} \quad (2.10)$$

We see that the transverse electric field is greatly enhanced, while the longitudinal field remains the same.

We also see that in addition to electric fields there arises a magnetic field equal to

$$\vec{B} = \vec{v}_0 \times \vec{E} \quad (2.11)$$

Fortunately, this magnetic field can be neglected. First, the magnetic field is of the same order as the electric one, but the Lorentz force acting on the dipole is suppressed as $v/c \ll 1$, where v is the relative speed of the dipole constituents, since we assume dipole to be a nonrelativistic system. Second, the magnetic field is the field with very short period and high frequency $\approx \kappa\gamma$. If we consider the classical motion in such periodic field, we shall come to the system of differential equations for the particle moving in combined oscillator-Lorentz force field, where we can use the Floque theorem. The Floque theorem [18] says that the system of the ordinary differential equations with periodic coefficients has the solution:

$$x(t) = A(t)C(t)$$

where B is the periodic matrix function and A is the diagonal matrix, with entries $\exp \lambda t$, the coefficients λ are called the eigenvalues of the problem. The matrixes are 6×6 and include both the 3 coordinates and its derivatives in time. Due to the energy conservation law and the fact that the classical energy of the oscillator is $\approx (dx/dt)^2 + \omega_0^2 x^2$, it is clear that all eigenvalues of the problem must be imaginary and appear in pairs (since the solution is real). Thus magnetic fields may contribute only to high frequency small fluctuations along the classical trajectory corresponding to the motion without the magnetic field. Consequently, it's effect can be safely neglected. Clearly the same is true for the quantum mechanical case, since the corresponding path integral is saturated by the classical trajectories.

We have seen that the fields that act on the oscillator in it's rest frame are time-dependent. In the eikonal approximation the coordinate of the bound state mass center is

$$z = v_0 t = v_0 T \gamma,$$

where T is the proper time in the oscillator frame, and γ is a Lorentz factor:

$$T = t/\gamma \quad (2.12)$$

Using the time and frequency transformation and field transformation laws we can write the correlation function of the potentials in the oscillator rest frame. The correlation function will become time-dependent:

$$\langle V(\vec{r}, T) V(\vec{r}', T') \rangle = A^2 \gamma^2 \exp(-\kappa q)/q \quad (2.13)$$

$$q = \sqrt{(x - x')^2 + (y - y')^2 + \gamma^2(z - z' + v_0(T - T'))^2} \quad (2.14)$$

In the latter equations x and y are the transverse coordinates that are the same in both reference frames, while z -s are the z -coordinates of the oscillator in the c.m. frame. Due to

the Lorentz contraction, we can neglect z dependence and therefore assume $z = z'$ (this can be done in Quantum Mechanics, where we neglect radiation). Recall that T and T' are the proper time of the oscillator. Thus we see that our problem of the moving oscillator wave packet has been reduced to the problem of the nonrelativistic oscillator at rest in the high frequency external electric field that is strong in the transverse plane, and it's longitudinal component has the same strength as in the media at rest, with the random field correlator (2.14).

In the small oscillator limit discussed above, the electric field can be considered as homogeneous, and we get the correlation function

$$\langle V(T)V(T') \rangle = A^2 \gamma^2 \exp(-\kappa q)/q \quad (2.15)$$

$$q = v_0 \gamma |T - T'| \approx \gamma |T - T'| \quad (2.16)$$

Here and below except in the γ factors we assume $v_0 = c$, and use the units where $c = 1$.

We shall argue below that the results obtained in the homogeneous field approximation hold qualitatively even for the large scale oscillator.

The eq. (2.14) holds also for the dipole, since in this case we work from the beginning in the homogeneous field approximation (2.7).

B. The General Formalism.

Now we have to consider the problem of the nonrelativistic oscillator in the random media, with the correlation function of the scalar potentials defined by eq. (2.15). This problem is very similar to the problem of the propagation of the electromagnetic wave in the random media characterized by the gaussian distribution function (see e.g. ref. [19]).

Our main goal is to find the level distribution of the oscillator at the moment of time t if at the 0 moment he was in the state characterized by the wave function $\Psi(\vec{r}, 0)$, or equivalently by a set of coefficients $a_n(0)$:

$$\Psi(\vec{r}, 0) = \sum a_n \Psi_n(\vec{r}) \quad (2.17)$$

Here Ψ_n are the wave functions of the stationary states of the pure oscillator without the random fields:

$$\Psi_n(x) = (M\omega_0/\pi)^{1/4} \sqrt{\frac{1}{2^n n!}} \exp(-M\omega_0 x^2/2) \quad (2.18)$$

(we shall choose the units where the Planck constant $\hbar = 1$). The probability that at the time t the oscillator will be at the level n is given by

$$P(n, t) = \langle |a_n(t)|^2 \rangle \quad (2.19)$$

The average is over all field configurations. The simplest way to calculate the latter average for the oscillator is to calculate the average product of the Green functions:

$$F(\vec{x}_0, \vec{y}_0, \vec{x}_1, \vec{y}_1, t) = \langle G(\vec{x}_0, \vec{x}_1, t) G^*(\vec{y}_0, \vec{y}_1, t) \rangle \quad (2.20)$$

Here G is the oscillator Green function in the external field V , that describes time evolution of the oscillator in the given realization of the field V . The average is taken over all external fields V with a Gaussian weight. Once we know the function $G(x_0, y_0; x_1, y_1, t)$, the time evolution of the a_n coefficients can be easily calculated:

$$|a_n(t)|^2 = \int d^3x_0 d^3y_0 d^3x_1 d^3y_1 F(x_0, y_0, x_1, y_1, t) \Psi(x) \Psi(y) \Psi_n(x') \Psi_n(y') \quad (2.21)$$

Here we assumed that at $t=0$ the oscillator was in the state described by the wave function $\Psi(x, 0)$.

One possible way to carry the calculations is to use the functional integrals method. The Green function of the 3D quantum oscillator is well known and can be represented as (see e.g. ref. [20], [21], [22] and references therein):

$$G(x, y, t) = \int dx(t) \exp(i \int_0^t \frac{1}{2} M(\dot{x}^2 - \omega_0^2 x^2)) ds \quad (2.22)$$

Here the integration is over all paths with

$$x(t) = x_1, x(0) = x_0 \quad (2.23)$$

Consider now the arbitrary wave packet. The full functional integral we are interested in is

$$\begin{aligned} F(x_0, y_0, x_1, y_1, t) &= \int dx(t) \int dy(t) \int dV(x, t) \exp(i(\frac{M}{2}((\dot{x})^2 - \omega_0^2 x^2) - V(x) \\ &\quad - (i\frac{M}{2}((\dot{y})^2 - \omega_0^2 y^2) - V(y))) \\ &\quad \times \exp(-\int_0^t \int_0^t ds ds' d^3x d^3y V(x, s) K(x, s; y, s') V(y, s')) \end{aligned} \quad (2.24)$$

The operator K is just the inverse of the correlation function of the potentials V , and is determined by the properties of the media. The integration is over all paths with

$$x(t) = x_1, y(t) = y_1; x(0) = x_0, y(0) = y_0 \quad (2.25)$$

Note that in the eq. (2.24) the integration over the oscillator paths is in Minkowski space, with the usual Minkowski time t , and i before the action. However, in the part of the functional integral connected with the integration over V there is no i before the action, and the integration is purely statistical one. This integration is analogous to Euclidean space functional integral in the Quantum Field Theory, and it is evident that it has the same nature as the coupling of oscillator to the finite temperature thermal bath.

It is very hard to carry out the exact functional integration in eq. (2.24) even though the corresponding functional integrals over V and x, y are Gaussian. Indeed, integration over V can be carried out. We can proceed in the standard way, as in the theory of the random fields, by looking for extremum V trajectory, and then substituting the result into the functional integral (2.24) that will be now the integral only over x, y . In order to carry

the integration, we write the term $V(x(t), t)$ in the action as $\int d^3x \delta(x - x(t)) V(x, t)$. Varying the action over $V(x, t)$, we obtain:

$$i\delta(x - x(t)) - i\delta(y - y(t)) = - \int_0^t K(x, t; x', t') V(x', t') \quad (2.26)$$

This is the equation for the extremal trajectory. The latter equation (except the factor i due to the fact that we carry the average in the Minkowski space) looks like the potential created by two opposite charges moving over the trajectories $x(t)$ and $y(t)$. The equation can be inverted giving

$$V(x, t) = -i \int_0^t ds (R(x, t; x(s), s) - R(x, t; y(s), s))/2 \quad (2.27)$$

Here R is the correlation function (2.14), that is inverse to the operator K . Substituting this expression for V in the functional integral (2.24) we get

$$F(x, y, x', y', t) = \int dx(t) \int dy(t) \exp i(M/2)(\dot{x}^2 - \dot{y}^2 - \omega_0 x^2 + \omega_0 y^2) - S_A(x(t), y(t)) \quad (2.28)$$

Here the effective action A is

$$\begin{aligned} S_A = & \int_0^t dt' \int_0^{t'} ds (R(x(t'), t'; x(s), s) + R(y(t'), t'; y(s), s) \\ & - R(x(t'), t'; y(s), s) - R(y(t'), t'; x(s), s)) \end{aligned} \quad (2.29)$$

Note that the above action looks quite similar to the famous Feynman action for the polaron [23]. It describes two particles with the singular interaction potential, that includes both interaction with its own trajectory and the interaction with the trajectory of the second particle-the situation quite similar to the problem of electrostatic dipole.

It is possible to deal with the latter action using mean field approximation or perturbation theory, however for fast particle the problem can be significantly simplified.

In order to simplify the problem further, first note that the main contribution into the action comes from the region $t' \approx s$, due to the exponential Yukawa cutoff in G . In the area $t' \approx s$, we can expand

$$\vec{x}(t') - \vec{x}(s) \approx -(d\vec{x}/dt')(t' - s).$$

Then the radius in the Yukawa correlation function 2.14 will have the form

$$q = \sqrt{(t' - s)^2 \gamma^2 (v_0^2 + (d\vec{x}/dt')^2)} \quad (2.30)$$

Since the system is nonrelativistic, the second term in the brackets in eq. (2.30) can be neglected, and the system reduces to the usual quantum mechanics, i.e. we can neglect both the longitudinal and the transverse self-motion. Concerning the interaction with the trajectory of the second particle, the contribution is clearly maximal when $x(t)$ is close to $y(t)$, and in this case also we can neglect the transverse motion. This means that for a relativistic particle in the first approximation we can neglect the space dependence of the electric field and assume that the system is in the homogeneous time dependent random electric field. In fact it was clear from the very beginning that we can take our electric

field as homogeneous. Indeed, if we look at the action of the electric field, we see that the term due to the space inhomogeneity contributes into the action as $\kappa^2 E^2$, relative to $\kappa^2 \gamma^2 E^2$ due to the time evolution, and thus can be neglected, even if the initial wave packet is not of the small radius. We shall see below, that the inhomogeneity of the electric field must be taken into account only at the times of the order of the minimum of two scales: $\gamma^{4/3}/(\omega_0^2 \kappa)^{1/3}$, $\gamma^2/\sqrt{(d-1/d\kappa\omega_0)}$, when the longitudinal expansion of the wave packet becomes significant—of the order of $1/\kappa$. Thus our results can be applied even for the more general case when $1/\kappa \leq 1/\omega_0$.

Concerning the small dipole, we deal with it in the approximation of the homogeneous electric field from the beginning, the field being equal to the field in the c.m. of the dipole.

Next, we need to deal with the Coulomb singularity of the Yukawa potential. We shall follow the line of ref. [21]. The contribution of the singular part of the interaction to the functional integral is zero: the probability of such configuration is zero. Moreover, it is known that such singularity really does not appear due to quantum fluctuations, and it's appearance is due to the quasiclassical method of the path integral calculation, since in the classical case there can be fall on the center. Consequently, if we are interested in the qualitative results, we can substitute the singular correlation function with a nonsingular one, omitting the coulomb part and dealing directly with the electric fields. As for the strength of the electric field we can take the strength of the field it is clear from eqs. (2.5),(2.7) that we have to take the field in the reference frame where the center of mass of the system is at rest.

Thus we come to the conclusion that both qualitatively in the leading eikonal approximation and quantitatively we can describe our model by an oscillator coupled to the homogeneous time dependent random electric field with the correlation function in the longitudinal direction given by

$$\langle E_z(t)E_z(s) \rangle = B^2 \exp(-\kappa\gamma v_0|T-s|) \quad (2.31)$$

and in the transverse plane with the correlation function

$$\langle E_i(t)E_j(s) \rangle = (B^2\gamma^2) \exp(-\kappa\gamma v_0|T-s|)\delta_{ij} \quad (2.32)$$

Here T and s denote the proper time (in the moving oscillator frame of reference), and the frequency is increased greatly by γ -a Lorentz factor. The corresponding potential is $V(x) = \vec{E}\vec{x}$, and the corresponding Lagrange density in the action can be rewritten as

$$\delta S_A = A^2 \int dt E(t)(\partial^2 + \kappa^2\gamma^2)E(t)dt \quad (2.33)$$

Note that the simplifications we did are very similar to the Feynman trick for polaron, where he substituted the complicated nonlinear action of a polaron by an action of the harmonic oscillator.

C. The Calculation of the Effective Action.

Now we shall calculate the 4-point function F in the above model. In order to average over the field configurations we shall use the action

$$S = A^2 \int ((\partial E)^2 + \kappa^2 \gamma^2 E^2) dt \quad (2.34)$$

The constant A is determined using the relation with the correlation function of the oscillator:

$$\langle E(t)E(t') \rangle = B^2(1, \gamma^2) \exp -(|t - t'| \kappa \gamma) \quad (2.35)$$

(the brackets depend on whether the field is in longitudinal or transverse direction). Then for the longitudinal field we have :

$$E_L^2(0) = B_L^2 = 1/(A_L^2(2\kappa\gamma)) \quad (2.36)$$

For the transverse field we have [21] :

$$E_T^2(0) = B_T^2 \gamma^2 = 1/(A_L^2(2\kappa\gamma)) \quad (2.37)$$

Here both electric fields E_L and E_T are the fields in the frame moving with the velocity $v_0 \approx c$ where the harmonic oscillator is at rest , so that B is the $\sqrt{E_L^2}(0)$, where index L means the laboratory system, and the time t here and below refers to the frame moving with the oscillator. Hence for the longitudinal field

$$A_L = 1/(B_L \gamma^{(1/2)} \sqrt{2}) \quad (2.38)$$

For the transverse field:

$$A_T = 1/(B_T \gamma^{(3/2)} \sqrt{2}) \quad (2.39)$$

Let us now repeat the derivation of the 4-point function G for the concrete example of the averaging weight E^2 . The derivation is slightly different from the previous chapter, since there we dealt with potential V and varied over it, while for the final simplified model we can deal directly with electric field and vary over it that simplifies the problem.

Due to the obvious factorization we can start from the one-dimensional problem. The whole calculation is carried in the rest frame of the moving oscillator.

More explicitly, the functional integral we need to calculate is

$$F(x, y, x', y', T) = \int dx(t) dy(t) dE(t) \exp iS(x, y, \vec{E}) - S(E) \quad (2.40)$$

Here $S(x, y, E)$ is the action

$$S = \int_0^T dt \frac{M}{2} (\dot{x}^2 - \omega_0^2 x^2 - \dot{y}^2 + \omega_0^2 y^2) - \vec{E} \vec{x} + \vec{E} \vec{y} \quad (2.41)$$

The statistical weight $S(E)$ is given by eq. (2.34). The boundary conditions for x, y are evident. Note that the first term in the functional integral (3.9) is multiplied by i while the second is not. We first will carry the integration over the random fields, and then consider the effective action for x and y. The integral over the electric field E is Gaussian and can be taken. We first need to vary the action S over the fields E and then substitute the result into the action. Varying over E we get:

$$i(-x_i(t) + y_i(t)) = A^2(\partial^2 - \kappa^2\gamma^2)E_i(t) \quad (2.42)$$

The latter equation can be immediately solved using the Green function of the operator $(\partial^2 - \kappa^2)$:

$$(\partial^2 - \kappa^2) \exp(-|\kappa t - t'|)/(2\kappa) = \delta(t - t') \quad (2.43)$$

Consequently, we write

$$E_i(t) = -(i/(2A^2\kappa\gamma)) \int_0^T \exp(-|t - s|)(x_i(s) - y_i(s))ds \quad (2.44)$$

We now substitute the above expression for E into the action in order to get effective action for x and y . We have for the terms that previously contained the electric field

$$S_2 = - \int_0^T \int_0^T dt ds \exp(-\kappa\gamma|t - s|)(x(t) - x(s))(y(t) - y(s))/(2A^2\kappa\gamma) \quad (2.45)$$

Note that the internal integration over s in the action goes up to T and not t , like in the Minkowski field case. The reason is that the averaging over field configurations takes part only after the concrete trajectory in $x(t)$ is realized. Physically, we first calculate functional integral for given E realization, and then average over all E .

We now have the effective action $S_1 + S_2$, where

$$S_1 = iM(\dot{x}^2 - \dot{y}^2 - \omega_0 x^2 + \omega_0 y^2)/2 \quad (2.46)$$

The calculation of the functional integral proceeds in the standard way. We seek the solution of the classical equation of motion, giving the exponent, and then calculate the preexponential factor.

Let us start from the equations of motion. We get

$$iM(\frac{\partial^2 x}{\partial t^2} + \omega_0 x) = - \int_0^T \exp(-\kappa\gamma|t - s|)(x(s) - y(s))/(A^2\kappa\gamma) \quad (2.47)$$

$$iM(\frac{\partial^2 y}{\partial t^2} + \omega_0 y) = - \int_0^T \exp(-\kappa\gamma|t - s|)(x(s) - y(s))/(A^2\kappa\gamma) \quad (2.48)$$

We see that the equations for x and y are identical, thus

$$\frac{\partial^2(x - y)}{\partial t^2} + \omega_0^2(x - y) = 0 \quad (2.49)$$

The boundary conditions are:

$$\delta_1 = x(T) - y(T) = x_1 - y_1; \delta_0 = x(0) - y(0) = x_0 - y_0 \quad (2.50)$$

The corresponding solution is evidently

$$x(t) - y(t) = (\delta_0 \sin(\omega_0(T - t)) + \delta_1 \sin \omega_0 t)/(\sin \omega T) \quad (2.51)$$

Once we know $x-y$, we can substitute it to eq. (2.48) and get the equation for y , which is just the equation for the oscillator under the action of the external imaginary force.

$$\begin{aligned} \frac{\partial^2 x}{dt^2} + \omega_0^2 x = & i/((MA^2\kappa\gamma)\sin(\omega_0 T)(\kappa^2\gamma^2 + \omega_0^2)) \\ & \times (\exp(-\kappa\gamma t)(\omega_0\delta_1 - \delta_0(\omega_0\cos(\omega_0 T) + \kappa\gamma\sin(\omega_0 T))) \\ & + \exp(-\kappa\gamma(T-t))(\delta_0\omega_0 - \delta_1(\omega_0\cos(\omega_0 T) + \kappa\gamma\sin(\omega_0 T))) + \\ & + 2\kappa\gamma(\delta_1\sin(\omega_0 t) - \delta_0\sin(\omega_0(t-T))) \end{aligned} \quad (2.52)$$

Note that the external force that appears in the r.h.s. of the eq. (2.52) has three parts. Two of them are exponentially suppressed and are different from zero only close to $t=0$ or $t=T$. However, the third term is not suppressed, and, moreover, is a resonance force applied to the harmonic oscillator with a frequency ω_0 . This is of course what was to be expected, since the random field distribution contains all possible frequencies, including the resonant one. The solution for x for given boundary conditions will be a sum of the solution of the homogeneous equation plus the solution of the nonhomogeneous one:

$$x(t) = \frac{1}{\omega_0} \int_0^t \sin(\omega_0(s-t))u(s)ds + x_0 \cos(\omega_0 t) + B \sin(\omega_0 t) \quad (2.53)$$

Here $u(s)$ is the r.h.s. of eq. (2.52). The coefficient $B(T)$ is determined from the boundary conditions.

The general solution of the nonhomogeneous equation is a sum of five terms:

$$x(t) = x_a(t) + x_b(t) + x_c(t) + x_d(t) + x_e(t) \quad (2.54)$$

The first term is due to the resonant part of the external field and is given by

$$x_a(t) = iL(\delta_1\kappa\gamma t \cos(\omega_0 t) - \delta_0\kappa\gamma t \cos(\omega_0(T-t)))/\omega_0 \quad (2.55)$$

The second part includes the terms that come from the integration of the resonant part that are not linearly enhanced as the term above:

$$x_b(t) = iL\frac{\kappa\gamma}{\omega_0^2}(\delta_0\sin(\omega_0 t)\cos(\omega_0 T) - \delta_1\sin(\omega_0 t)) \quad (2.56)$$

Here

$$L = 1/(A^2\kappa\gamma)M\sin(\omega_0 T)(\kappa^2\gamma^2 + \omega_0^2)$$

The third part comes from the integration of the nonresonant force, but is not exponentially suppressed, however it is suppressed by additional power of $\kappa\gamma$ relative to the previous term:

$$x_c(t) = (iL/(\kappa\gamma^2 + \omega_0^2))(\omega_0\delta_1 - \delta_0(\omega_0\cos(\omega_0 T) + \kappa\gamma\sin(\omega_0 T)))(\omega\cos(\omega_0 t) - \kappa\gamma\sin(\omega_0 t)) \quad (2.57)$$

Finally the 4th term is the exponentially suppressed part coming from the nonresonant part of the external force:

$$\begin{aligned}
x_d(t) = & (iL/(\kappa\gamma^2 + \omega_0^2))(-\omega_0(\omega_0\delta_1 - \delta_0(\omega_0 \cos(\omega_0 T) + \kappa\gamma \sin(\omega_0 T))) \exp(-\kappa\gamma t) \\
& + (\delta_0\omega - \delta_1(\omega \cos(\omega_0 T) + \kappa\gamma \sin(\omega_0 T))) \\
& (\exp(-\kappa\gamma T)(\kappa\gamma \sin(\omega_0 t) + \omega_0 \cos(\omega t)) - \omega \exp(-((T - t))))
\end{aligned} \tag{2.58}$$

Finally there is a solution of the homogeneous equation that can be rewritten as:

$$\begin{aligned}
x_e(t) = & x_0 \cos(\omega_0 t) + (x_1 - x_a(T) - x_b(T) - x_c(T) - x_d(T) \\
& - x_e(T) - x_0 \cos(\omega_0 T)) \sin(\omega_0 t) / \sin(\omega_0 T)
\end{aligned} \tag{2.59}$$

Note that if we are interested in time scales

$$t_L \geq 1/\kappa$$

we can neglect the terms that are exponentially suppressed. The terms in x_b have the same structure essentially as in $x_b(t)$ but are enhanced by at least $\kappa\gamma/\omega_0$. Hence we can write for $t_L \geq 1/\kappa$

$$x(t) = x_a(t) + x_b(t) + x_e(t) \tag{2.60}$$

Here in $x_e(t)$ we can in the expression for $B(T)$ neglect all the terms except coming from $x_a(T) + x_b(T) + x_0 \cos(\omega_0 T)$. Then for $B(T)$ we have

$$\begin{aligned}
B(T) = & (x_1 - x_0 \cos(\omega_0 T) - iL\kappa\gamma(T(\delta_1 \cos(\omega_0 T) - \delta_0) + \sin(\omega T)(\delta_0 \cos(\omega_0 T) - \delta_1)/\omega_0) / \sin(\omega_0 T) \\
& (2.61)
\end{aligned}$$

We see that the classical trajectory of the oscillator in random field corresponds to the sum of two terms: first, the usual harmonic oscillations (due to the homogenic part of the solution), second the linear expansion (although suppressed as $1/\gamma$) in the imaginary direction .

We now calculate the action as the function of the boundary conditions using the solution (2.60). Due to the equations of motion the action is just

$$i(\dot{x}(T)x_1 - \dot{y}(T)y_1 - x_0\dot{x}(0) + \dot{y}(0)y_0), \tag{2.62}$$

The other terms in the action disappear due to the equations of motion. It is easy to obtain:

$$\begin{aligned}
S = & \frac{M\omega_0}{2}((x_1^2 - y_1^2 - x_0^2 - y_0^2) \cot(\omega_0 T)) - 2(x_0x_1 - y_0y_1) / \sin(\omega_0 T) \\
& + i(L\kappa\gamma / \sin(\omega_0 T))((\delta_1^2 + \delta_0^2)(\omega T - \sin(2\omega T)/2) + \\
& + 2\delta_0\delta_1(\sin(\omega_0 T) - \omega_0 T \cos(\omega_0 T))
\end{aligned} \tag{2.63}$$

This action gives us the transition operator as a function of boundary conditions. Note that it contains three types of terms: 1) real part; 2) Imaginary part with linearly enhanced terms due to quantum fluctuations 3) Imaginary part with only trigonometric terms. Note that we neglected for all the terms that are suppressed by $\exp(-T)$ and by the powers of γ . The transition operator in the quasiclassical approximation will be

$$F = \exp(iS) \tag{2.64}$$

We need now to consider the preexponential factor, and normalize to the evolution operator of the harmonic oscillator without electric fields. In order to calculate the preexponent we need to expand:

$$x(t) = x_{cl}(t) + \sum_0^{\infty} c_n \sin(2\pi n t/T) \quad (2.65)$$

$$y(t) = y_{cl}(t) + \sum_0^{\infty} b_n \sin(2\pi n t/T) \quad (2.66)$$

Since the cross terms with x_{cl} evidently cancel we need to consider the integral in the action due to the sum over n . It is equal to:

$$\begin{aligned} P = & \prod \exp(iM(b_n^2 - c_n^2)((2\pi n/T)^2 + \omega_0^2)) \\ & - \frac{1}{2A^2\kappa\gamma} \sum (c_n - b_n)(c_m - b_m) \int_0^T \int_0^T \sin(2\pi n s/T) \\ & \times \exp(-\kappa\gamma|t - s| \sin(2\pi m t/T)) dt ds \end{aligned} \quad (2.67)$$

We can now move to the integration over the variables $u_n = b_n - c_n$ and $v_n = b_n + c_n$. Then we evidently get after the integration over v_n the product of delta-functions

$$\prod \delta(u_n((2\pi n/T)^2 + \omega_0^2))$$

Then we can evidently let $u_n = 0$. The preexponential factor P will be the same as the corresponding factor for the oscillator without the electric field which is evidently:

$$\prod_{n=0}^{n=\infty} \frac{1}{(2\pi n/T)^2 + \omega_0^2} = \sin(\omega_0 T) \quad (2.68)$$

Now we can calculate the transition operator G , whose matrix elements give the transition probabilities we are looking for. We obtain

$$\begin{aligned} F(x_0, y_0; x_1, y_1; T) = & \frac{1}{\sin(\omega_0 T)} \exp((iM\omega/2) \left(\frac{1}{\sin(\omega_0 T)} ((x_0^2 + x_1^2 - y_0^2 - y_1^2) \right. \\ & \left. \cos(\omega_0 T) - 2(x_0 x_1 - y_0 y_1)) \right)) \\ & \times \exp - (ML\kappa\gamma \frac{1}{\sin(\omega_0 T)} (\delta_1^2 + \delta_0^2) (\omega_0 T - \sin(2\omega_0 T)/2) \\ & + 2\delta_0 \delta_1 (\sin(\omega_0 T) - \omega_0 T \cos(\omega_0 T))) \end{aligned} \quad (2.69)$$

Note that the first term is just the usual evolution operator for the harmonic oscillator, while the second term contains the action of the random field. The operator is written in terms of the proper time, in the oscillator reference frame. In order to work in the laboratory frame we need to substitute $T = t_L/\gamma$, where t_L is the laboratory time. We can use the latter transition operator to calculate transition amplitudes as the functions of time.

D. Transition Amplitudes.

Once we know the transition operator (2.69) we can calculate the evolution of the initial distribution. If in the initial state $t=0$ the oscillator is in the state k , the probability that after time t the oscillator will be in the state k is

$$|a_n^k(T)|^2 = \int dx_1 dx_0 dy_1 dy_0 H_n(x_1) H_n(y_1) H_k(x_0) H_k(y_0) \exp - (M\omega_0/2)(x_0^2 + y_0^2 + x_1^2 + y_1^2) F(x_0, y_0, x_1, y_1; T) \quad (2.70)$$

Here N_n is the normalization factor:

$$N_n = (M\omega_0/\pi)^{1/4} \sqrt{1/(2^n n!)} \quad (2.71)$$

In order to carry the actual calculation we need to go to the variables $s_0 = x_0 + y_0$, $s_1 = x_1 + y_1$; δ_0, δ_1 and carry out the corresponding gaussian integral over the four variables. Note that due to the cross-term between $\delta_0 \delta_1$ in the transition operator F we have

$$| \langle a \rangle |^2 \neq \langle |a|^2 \rangle \quad (2.72)$$

The easiest example to calculate is the transition probability $|a_0^0|^2$ for the oscillator entering the media in the initial state, and then staying in the initial state. The corresponding gaussian integral is

$$|a_0^0(T)|^2 = 1/\sqrt{H} \quad (2.73)$$

where H is given by

$$H = 1 + 2\omega_0 D t_L + 4D^2(\omega_0^2 t_L^2 - \gamma^2 \sin^2(\omega_0 t_L/\gamma)) \quad (2.74)$$

Here

$$D_L = 2B_L^2 \kappa / (M\omega_0^2(\kappa^2 \gamma^2 + \omega_0^2)) \quad (2.75)$$

for the longitudinal motion and

$$D_T = 2B_T^2 \kappa \gamma^2 / (M\omega_0^2(\kappa^2 \gamma^2 + \omega_0^2)) \quad (2.76)$$

for the transverse motion. We see that for $t_L \gg \gamma/\omega$ the dependence is

$$\approx 1/(D\omega_0 t_L)$$

The result for the arbitrary n, k is given by the derivative:

$$\begin{aligned} \langle |a_k^n(T)|^2 \rangle &= H_n((\partial_{\lambda_1} + \partial_{\lambda_2})/2) \times H_n((\partial_{\lambda_2} - \partial_{\lambda_1})/2) \times H_k((\partial_{\mu_2} - \partial_{\mu_1})/2) \\ &\quad H_k((\partial_{\mu_2} + \partial_{\mu_1})/2) I(T, \mu_1 = 0, \mu_2 = 0, \lambda_1 = 0, \lambda_2 = 0) \end{aligned} \quad (2.77)$$

Here the integral I is

$$I = \int ds_0 ds_1 d\delta_0 d\delta_1 \exp(iS + \mu_1 s_0 + \mu_2 \delta_0 + \lambda_1 \delta_0 + \lambda_2 \delta_1 - M\omega_0(\delta_1^2 + \delta_0^2 + s_1^2 + s_0^2)/4) / \sin(\omega_0 T) \quad (2.78)$$

The integral I is a gaussian integral that can be easily taken:

$$I = \frac{1}{\sqrt{H}} \exp(-Z/H - (\lambda_1^2 + \mu_1^2)/M\omega_0)) \quad (2.79)$$

Here

$$\begin{aligned} Z = & (M\omega(1 + D(\omega T - \sin(2\omega T)/2))(\mu_2^2 + \lambda_2^2) \\ & - (M\omega(1 + D(\omega T + \sin(2\omega T)/2))(\mu_2^2 + \lambda_2^2) \\ & + 2iDM\omega \sin^2(\omega T)(\mu_1\mu_2 + \lambda_1\lambda_2) \\ & - 2i(\lambda_2\mu_1 + \lambda_1\mu_2) \sin(\omega T)M\omega(1 + D\omega T) + 2\mu_1\lambda_1 M\omega(\cos(\omega T) + D\omega T + D \sin(\omega T)) \\ & + \mu_2\lambda_2 M\omega(\cos(\omega T) + D\omega T - D \sin(\omega T)))/2H \end{aligned} \quad (2.80)$$

The problem is of course 3Dimensional, and the explicit expression for 3D coefficients are just a direct product of one-dimensional coefficients in $x - y$ plane and in z direction

$$|a_{k_1 k_2 k_3}^{n_1 n_2 n_3} = \prod_{i=1,2} a_{k_i}^{n_i} \delta_{k_3}^{n_3} \quad (2.81)$$

Here we took into account that for sufficiently big γ the transverse fields are strongly enhanced and we need only to take into account the transverse transitions.

The evolution of the arbitrary initial distribution (2.17) can be calculated in the same way.

III. EVOLUTION OF THE ARBITRARY WAVE PACKETS.

In the previous section we considered the time evolution of the arbitrary initial states. In this section we shall consider the evolution of the small wave packet that has at $t=0$ the form in the transverse plane:

$$\Psi(y) = (2dM\omega_0/\pi)^{1/4} \exp(-dM\omega_0 y^2) \quad (3.1)$$

We first calculate explicitly the density matrix

$$\rho(x, y) = \langle \Psi(x, t)^* \Psi(y, t) \rangle$$

where the average means the averaging over all random field configurations, and then use this expression to calculate the probabilities. Substituting eq. (3.1) into the equations of the previous section, we get for the density matrix

$$\begin{aligned} \rho(x, y, t) = & \exp(-\delta^2(d/16 + D^2 d(\omega_0^2 t^2 - \gamma^2 \sin^2(\omega_0 t/\gamma) + (\omega t + \gamma \sin(2\omega_0 t/\gamma)/2.)D/4 \\ & + d^2 D(\omega_0 t - \sin(2\omega_0 t/\gamma)/2) - ds^2/16 + 2i\delta s((d^2 - 1) \sin(2\omega_0 t/\gamma)/32 \\ & + Nd\gamma \sin^2(\omega_0 t/\gamma)/4))/D(t))/\sqrt{D(t)} \end{aligned} \quad (3.2)$$

Here D is given by eq. (2.76) and the function $D(t)$ is given by

$$D(t) = (d^2 \sin(\omega_0 t / \gamma)^2 + \cos(\omega_0 t / \gamma))^2 / 4 + D d(\omega_0 t - \gamma \sin(2\omega_0 t / \gamma) / 2) \quad (3.3)$$

We can determine density radius of the dipole/wave packet. The density radius characterizes the decrease of $\langle |\Psi(x, t)|^2 \rangle$ as a function of x^2 . Note, that this is completely different physical quantity than the effective radius of the dipole/wave packet, that determines the effective cross-sections.

The expansion of the wave packet density is determined by $\rho(x, x) = \langle \Psi(x, t)^* \Psi(x, t) \rangle$

$$x^2 \approx D(t)/d = (d \sin(\omega_0 t / \gamma)^2 + (1/d) \cos(\omega_0 t / \gamma)) / 4 + D(\omega_0 t - \gamma \sin(\omega_0 t / \gamma) / 2) \quad (3.4)$$

Note that for the small $t \ll \gamma / \omega$ we have

$$x^2(t) \approx (1/d + \frac{3}{4}(d - 1/d)\omega_0 t^2 / \gamma^2 + 16D\omega_0^2 t^3 / \gamma^2) / 4 \quad (3.5)$$

We see that there are three pieces in the latter equation: first, the initial radius of the wave packet, second-the geometrical factor, that is equal to zero when the particle is in the ground state, and is bigger, the smaller is the wave packet radius (in correspondence with the uncertainty principle). This factor describes the wave-packet diffusion, that is present when there are no interactions. Finally, the third term is wave-packet radius independent and describes the density expansion due to a strong field. We see that the latter for small t goes as t^3 , and for the case of the ground state $d = 1$ this is the only term that contributes to the expansion.

Before the time t_L reaches $t_k \approx (d - 1/d) / (\omega_0 D)$ the density radius squared increases according to a square law, while for times $\geq \gamma / \omega_0$ it increases according to the linear law.

We can determine, till what time scale we can consider the field \vec{E} as homogeneous. It is well known in the theory of random systems [24] that the field can be considered homogeneous if the inhomogeneity scale in transverse direction is much bigger than time (and z) correlation scale. In our case the latter scales are $1/\kappa$ and $1/(\kappa\gamma)$ correspondingly. However, due to the longitudinal diffusion the longitudinal-time correlation scale increases. Due to the fact that the longitudinal electric field does not change strength due to Lorentz boost, the expansion for the small packet in the longitudinal direction is determined by the geometrical factor. For the big packet with the radius of the ground state or bigger the expansion rate will still be determined by the diffusion. The longitudinal scale will reach the radius $1/\kappa$ at the minimum of the times $\gamma^2 / \sqrt{(d - 1/d)\omega_0 \kappa}$, $\gamma^{4/3} / (\omega_0^2 \kappa)^{1/3}$. This means that for all realistic values of d and for sufficiently large γ , our results, derived for the homogeneous fields, certainly hold up to the times of $\approx \gamma / \omega$, that is the characteristic time in the interaction of the dipole and the target.

For the case of the oscillator the same result can be obtained in the different way, using the Erenfest theorem. Indeed

$$\frac{\partial^2 \vec{x}}{\partial x^2} + \omega^2 \vec{x} = \vec{E}(t)/M \quad (3.6)$$

This operator equation is easily solved:

$$\vec{x}(t) = A \cos(\omega t_L/\gamma) + \int_0^t ds \sin(\omega(t-s)) E(s) \vec{M} \quad (3.7)$$

Then

$$\langle \rho^2(t) \rangle = (B^2 \kappa \gamma) \int_0^t \int_0^t ds ds' \sin(\omega(t-s)) \sin \omega(t-s') \langle E_z(s) E_z(s') \rangle \quad (3.8)$$

for longitudinal diffusion, and

$$\langle x^2(t) \rangle = (B^2 \gamma^2 \kappa \gamma) \int_0^t \int_0^t ds ds' \sin(\omega(t-s)) \sin \omega(t-s') \langle E_T(s) E_T(s') \rangle \quad (3.9)$$

for transverse diffusion. In this way we obtain the law of expansion:

$$\langle \rho^2(t) \rangle \approx B^2 ((2t_L - \gamma \sin(2\omega t/\gamma)/(2\omega)) \times \kappa^2 \gamma^2 / (\kappa^2 \gamma^2 + \omega^2)) \quad (3.10)$$

We depicted the characteristic dependence of the square radius of the oscillator in the initial ground state and in the initial small wave packet state in figures 8,9,10 and 11,12,13 respectively. Note that in this derivation it is irrelevant if we consider classical or quantum problem. The coincidence of the results of two approaches is of course due to the Erenfest theorem.

Once we know the laws of quantum expansion we can calculate the probability distribution for the system to be in the n-th state. The probability is given by

$$\langle |a_n(t)|^2 \rangle = S(t)^{n/2} / R(t)^{(n+1)/2} P_n(Q(t)/(2D(t)\sqrt{S(t)R(t)})) \quad (3.11)$$

Here

$$Q(t) = D^2 d(\omega^2 t^2 - \gamma^2 \sin(\omega t/\gamma)^2) + D(\omega t + \gamma \sin(2\omega t/\gamma)/2)/4 + N d^2((\omega t - \sin(2\omega t/\gamma)/2)/4). \quad (3.12)$$

P_n are the Legendre Polynomials, and $D(t)$ is given by eq.(3.3);

$$\begin{aligned} S(t) = & -1/4 - (D^2 d^2(\omega^2 t^2 - \sin^2(\omega t/\gamma) \cos^2(\omega t/\gamma) + d^2/16 \\ & + Dd(\omega t + \gamma \sin(4 * \omega t/\gamma)/4)/4 + Dd^3(\omega t - \sin(4\omega t/\gamma)/4.)/4 \\ & + (d^2 - 1)^2 \sin(2\omega t/\gamma)^2/64)/(4D(t)^2)) \\ & (d/8 + D^2 d(\omega_0^2 t^2 - \gamma^2 \sin^2(\omega t/\gamma)) + D(\omega t + \gamma \sin(2\omega t/\gamma)/2)/4 \\ & D d^2(\omega t - \sin(2\omega t/\gamma)/2)/4.)/D(t) \end{aligned} \quad (3.13)$$

$$\begin{aligned} R(t) = & +1/4 + (D^2 d^2(\omega^2 t^2 - \sin^2(\omega t/\gamma) \cos^2(\omega t/\gamma) + d^2/16 \\ & + Dd(\omega t + \gamma \sin(4 * \omega t/\gamma)/4)/4 + Dd^3(\omega t - \sin(4\omega t/\gamma)/4.)/4 \\ & + (d^2 - 1)^2 \sin(2\omega t/\gamma)^2/64)/(4D(t)^2)) \\ & (d/8 + D^2 d(\omega_0^2 t^2 - \gamma^2 \sin^2(\omega t/\gamma)) + D(\omega t + \gamma \sin(2\omega t/\gamma)/2)/4 \\ & D d^2(\omega t - \sin(2\omega t/\gamma)/2)/4.)/D(t) \end{aligned} \quad (3.14)$$

The sample graphs are given below (see figures 1,2,3).

IV. THE GROUND STATE EVOLUTION.

Consider now the oscillator entering the random media in the ground state. Then we can put $d=1$ in the formulae of the preceding chapter and all results will be significantly simplified:

$$R(t) = H(t)/(4D(t)), \quad S(t) = D^2(\omega_0^2 t^2 - \gamma^2 \sin(\omega_0 t/\gamma)^2)/D(t) \quad (4.1)$$

The probability is given by

$$\begin{aligned} \langle |a_n(t)|^2 \rangle &= ((2D)^2(\omega_0^2 t^2 - \gamma^2 \sin(\omega_0 t/\gamma)^2)^{n/2} / (H(t))^{(n+1)/2}) \\ &\times P_n(D\sqrt{(\omega_0^2 t^2 - \gamma^2 \sin(\omega_0 t/\gamma)^2)/H(t)}) \end{aligned} \quad (4.2)$$

The sample graphs are given below (see figs. 4,5,6).

These results can be applied to the simplest model of charmonium moving through the Quark-Gluon plasma. This picture of charmonium is of course not realistic, since it does not contain absorption [2,14,25], coulomb interactions [10,11], color and radiation. It is given here for illustrative purposes only. The more realistic picture of charmonium will be discussed elsewhere. In our model the charmonium enters the media (say quark-gluon plasma or nuclear matter) as the dipole in the ground state. The charmonium is scattered by the random electric fields in the media and is excited to the states with $N = 2$ or higher states, where it decays via $D\bar{D}$ pair decay mechanism. We assume for simplicity that the decay probability is one, i.e. once the charmonium crosses the $D\bar{D}$ threshold [10,11], the total probability that charmonium will still be charmonium and not decay will be given by the sum

$$P(T) = \sum_{i=0, j=0, k=0}^{i+j+k \leq N_0} |a(0 \rightarrow ijk)|^2 \quad (4.3)$$

Here N_0 is the energy level corresponding to the $D\bar{D}$ threshold. It is enough to take into account only transitions in transverse directions, so the sum is given by

$$P(t) = \sum_{p+s \leq N_0} |a_0^p a_0^s|^2 \quad (4.4)$$

The result is depicted in figure 7. In the calculations we have taken the realistic $\omega_0 = 1$ GeV. For the realistic κ we take 0.12 GeV, for $\gamma \approx 30$ (that corresponds to quick charmonium produced in the Fermilab experiment), and for the field B we take the field in the QCD vacuum [30], that is say 0.11 GeV², and for illustrative purposes we took $N_0 = 2$.

Our results imply that this mechanism can make a significant contribution in the dissociation of the rapid charmonium in the QCD media.

We also did not take into account that there exist, for the small wave packet, backward transitions from higher levels to low ones, that must be discarded in the realistic model (since charmonium will dissociate). This question will be dealt separately.

V. CONCLUSION.

We have studied the qualitative influence of the random media on the relativistic propagation of the nonrelativistic dipoles and wave packets with internal interaction (see eqs. (3.2),(3.5)). We showed that the media: 1) enhances the expansion rate and leads to the new regime of the superdiffusion expansion at the rate $\rho^2 \approx t^3$ for a ground state and for the small wave packet for a sufficiently large time intervals if a field is sufficiently large; 2) leads to the diffusion over levels, leading to new types of diffractive events.

We have seen that the wave packet expansion rate depends on the field strength and can be used to study the properties of the media. Moreover, our results strongly suggest that the energy losses and radiation of the dipole or wave packet propagating through the random media will strongly depend on the media (compare with the corresponding calculation for a color parton [28]), and can be used for the diagnostics of the quark-gluon plasma. Our results also imply an additional mechanism of the charmonium output suppression relative to the known ones [25,27].

Our results imply the possibility of an additional mechanism of dissociation of rapid charmonium by excitation to the higher levels due to scattering on the random fields, and consequent dissociation into $D\bar{D}$ pairs.

We have calculated the transition operator (2.69),(2.63)) for the oscillator, and calculated the time dependence of the probabilities $P_n(t)$ of the oscillator occupying the n -th level if it was in some initial state. The corresponding formula is given by eqs. (3.12),(2.76),(3.13),(3.14),(3.11). We used these results to study the simple illustrative model of charmonium, and were able to determine how many charmoniums will stay after time t (or, equivalently, if a charmonium beam goes through the shell of the width $L = v_0 t$) (see eqs. (4.2), (4.3)).

The results obtained in this paper were derived for a particular case of the moving oscillator. We expect that the same results still hold, at least qualitatively, for more general system, such as a Coulomb one. and. Indeed, the mechanism of superdiffusion expansion is the pick-up by the system of the resonance frequencies from the media field frequency spectrum. In the system other than oscillator, say in the Coulomb system, it will pick the frequencies corresponding to the distances between nearby levels, and consequently will quickly expand, as in the simple oscillator case. It will be very interesting and necessary for the study of realistic systems to take into account absorption [2,14].

We have calculated the effective action and transition operator (2.45) for the propagation of the general relativistic dipole in the media. This action, obtained after integration out of the electric field is quite general, and can be used for the arbitrary bound systems. However, for the cases other than harmonic oscillator it is hard to carry the exact calculations and one may resort to numerical methods.

It will be very interesting to use our method to study a number of problems not considered here: the propagation of positronium through amorphous media and it's ionization rate, and the more realistic charmonium model that includes color and absorption, as well as to the detailed analysis of the influence of the nonhomogeneity. It will be also very interesting to carry the calculation of the radiation loss and it's influence on the dynamics of the dipole. At this point, due to the appearance of the traces of the several color matrices, we shall expect real technical difference between calculations in the usual plasma and quark gluon

plasma.

ACKNOWLEDGMENTS

The author is indebted to Prof. L. Frankfurt for numerous very fruitful discussions and careful reading the paper. The author is also thanks Dr. A. Kamenev for useful discussions.

APPENDIX A: THE PLASMA MODEL AND IT'S STATISTICAL DESCRIPTION.

The aim of this section is to connect the usual language of the ref. [15] model with the language of the statistical mechanics of the random media.

Let us recall briefly the model of ([15]). The model describes the QCD media as the usual Debye plasma, with a potential

$$V_i^a(\vec{q}) = g(T_i^a)_{cc'} \exp(-i\vec{q}\vec{x}_i)/(\vec{q}^2 + \mu^2) \quad (\text{A1})$$

Here μ is the Debye screening mass. The parton-parton interactions are proportional to $tr(T_i^a T_j^b)$. The averaged potential vanishes everywhere since $\langle V_i^a \rangle \sim Tr T_i^a = 0$, while the parton-parton interactions are proportional to

$$Tr(T_i^a T_j^b) = \delta_{ab} \delta_{ij} (d_i/d_A) C_{2i} \quad (\text{A2})$$

Here d_i is the dimension of the SU(N) representation, C_{2i} is the second Casimir, $((N^2-1)/(2N))$ for quarks ($d_i = N$) and N for gluons ($d_i = N^2 - 1$). Thus after averaging we see that such model looks like multicomponent plasma, with a particular interacting charge for each set of color partons, with different types of partons not interacting in the leading order. Thus, we expect that apart from some numerical factors, connected with the Casimirs, the propagation of the wave packet in such a plasma will be qualitatively the same as it's propagation through the usual electromagnetic plasma described in the article. The true color effects will appear only when we shall take into account multiparticle correlations.

We now turn to the description of the charged plasma in the language of the statistical mechanics ([23]). In order to do it, we can just rewrite the statistical sum of the plasma as a Euclidean Functional Integral. The choice of the action must be such that it reproduces the correlation function (2.2). Clearly, the functional integral is Gaussian and can be represented as

$$Z = \int dE \exp(- \int d^3q V(q^2 + \kappa^2) V) \quad (\text{A3})$$

Moving into the oscillator rest frame and taking into account that the time derivative term will be dominant, we come to the action (2.62). This description is just the usual language of the statistical mechanics of the random media.

APPENDIX B: LORENTZ TRANSFORMATIONS AND CENTER OF MASS MOTION.

Let us introduce the center of mass coordinates (for simplicity let us assume that both particles have the same mass) v and the relative motion coordinate u :

$$\vec{v} = (\vec{r}_1 + \vec{r}_2)/2 \quad (\text{B1})$$

$$\vec{u} = (\vec{r}_1 - \vec{r}_2)/2 \quad (\text{B2})$$

$$(\text{B3})$$

We shall denote M the reduced mass of the oscillator in c.m. reference frame. Then the Lagrangian can be written as a sum:

$$L = \int ds(v) + L_{\text{n.r.}}(\vec{u}) \quad (\text{B4})$$

Indeed, the full Lagrangian is

$$L = \int ds(r_1) + ds(r_2) - (V(\vec{r}_1) - V(\vec{r}_2) + V(\vec{u}))dt \quad (\text{B5})$$

Here

$$ds(\vec{r}) = \sqrt{1 - \left(\frac{\partial \vec{r}}{\partial t}\right)^2} dt \quad (\text{B6})$$

We then write:

$$ds(\vec{r}_1) = \sqrt{1 - \left(\frac{\partial(\vec{u} + \vec{v})}{\partial t}\right)^2} dt \quad (\text{B7})$$

$$ds(\vec{r}_1) = \sqrt{1 - \left(\frac{\partial(\vec{u} - \vec{v})}{\partial t}\right)^2} dt \quad (\text{B8})$$

In these two expressions we expand their r.h.s. in Taylor series in u around v . Then we immediately obtain:

$$ds(r_1) + ds(r_2) = ds(v) + \int dt(\dot{\vec{v}}^2)/(\sqrt{1 - \dot{\vec{u}}^2}/2) = ds(v) + \int dT(\dot{\vec{v}}^2)/2 \quad (\text{B9})$$

Here T is the oscillator proper time. Adding the interaction term to the eq. (B9) we immediately obtain the decomposition (B4).

Let us now discuss briefly the Lorentz transformation [17]. In the laboratory system we have only the electric field described by the static potential V . The Lorentz transformation says:

$$V = (V' - vA'_3)\gamma \quad A_3 = (A'_3 - vV')\gamma \quad (\text{B10})$$

In the laboratory frame $A_3 = 0$, hence $A'_3 = vV'$. Substituting the latter formula in eq. B10 we get

$$V = V'(1 - v^2)\gamma = V'\gamma \quad (\text{B11})$$

Finally we get

$$V' = V\gamma \quad A'_3 = vV' \quad (\text{B12})$$

Differentiating the latter equations in the moving reference frame we obtain the transformation laws in the text.

APPENDIX C: THE CALCULATION OF THE INTEGRAL.

Here for completeness we shall write the formulae for the double integral used in the text:

$$\begin{aligned} I = & \int_{-\infty}^{\infty} dx \int_{-\infty}^{\infty} dy \exp(-Qx^2 - Q^*y^2 - 2Pxy) H_n(x) H_n(y) / (2^n n!) \\ & (Q + Q^* + P^2 - 1 - |Q^2|)^{n/2} / (|Q^2| - P^2)^{(n+1)/2} \\ & P_n(-P/\sqrt{(|Q^2| - P^2)(Q + Q^* - |Q^2| + P^2 - 1)}) \end{aligned} \quad (\text{C1})$$

The integral was calculated using ref. [31] The Legendre Polynoms are defined as [32]:

$$2^n n! P_n(x) = \frac{d^n}{dx^n} (x^2 - 1)^n \quad (\text{C2})$$

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FIGURES

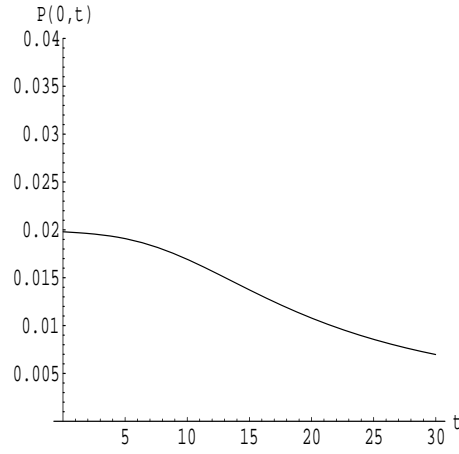


FIG. 1. Probability of the oscillator that is a small wave packet with radius $a=0.1$ (here and below in the ground state radius units) to go from the ground to the tenth state as the function of time. We use here and below: $\omega_0 = 1$ GeV, $M = 1$ GeV-the reduced mass of the sample quarkonium, $\gamma = 30$, and $\kappa = 0.12$ GeV; $B^2 = 0.012$ GeV⁴.

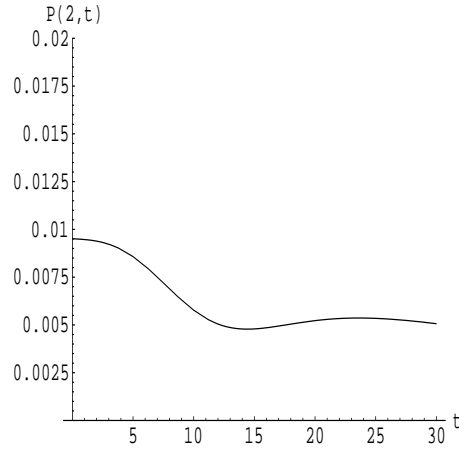


FIG. 2. Probability of the small oscillator wave packet with radius $a=0.1$ to be in the second state at the time t .

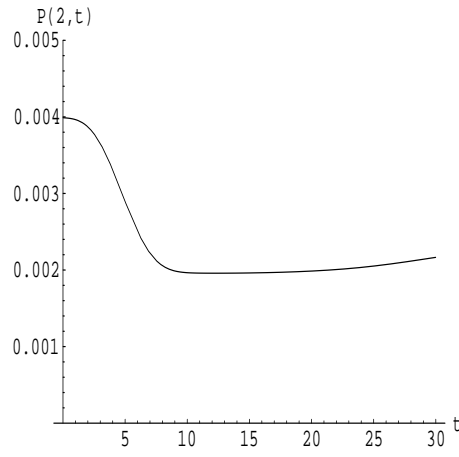


FIG. 3. Probability of the small oscillator wave packet with radius $a=0.1$ to be in the tenth state as the function of time.

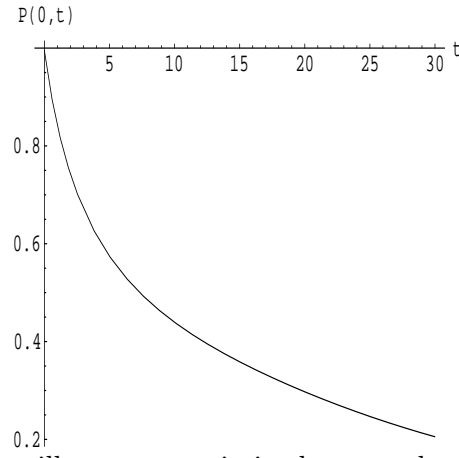


FIG. 4. Probability of the oscillator to remain in the ground state, if it enters the media in the ground state, as a function of time.

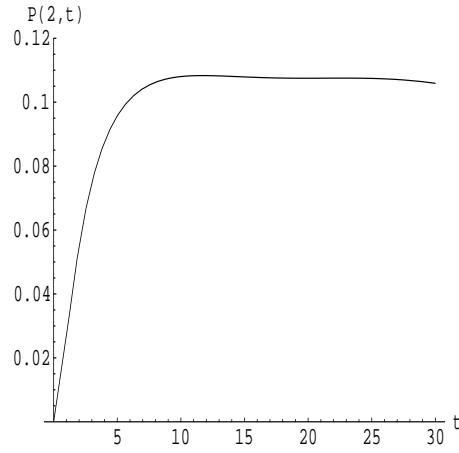


FIG. 5. Probability of the oscillator to go from the ground to the second state as the function of time.

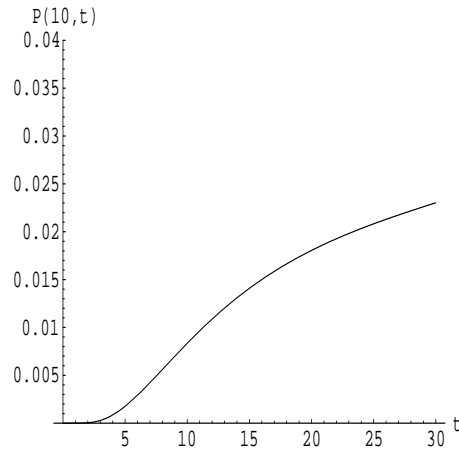


FIG. 6. Probability of the oscillator to go from the ground to the tenth state as the function of time.

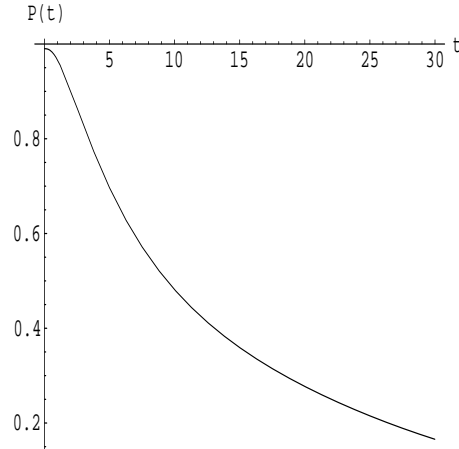


FIG. 7. Probability of the charmonium entering the media in the ground state to remain charmonium at time t .

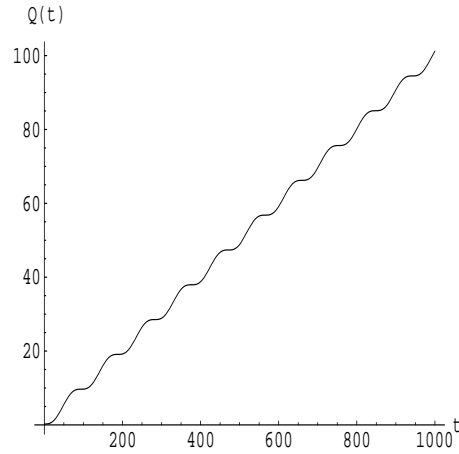


FIG. 8. Density radius of 1D oscillator in the random media as a function of time for big time scales, if initially it was in the ground state. Here and below $Q(t) = x^2(t)$

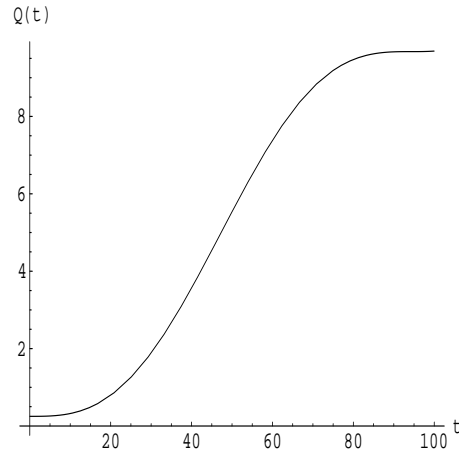


FIG. 9. Density radius of 1D oscillator in the random media as a function of time for average time scales, if initially it was in the ground state.

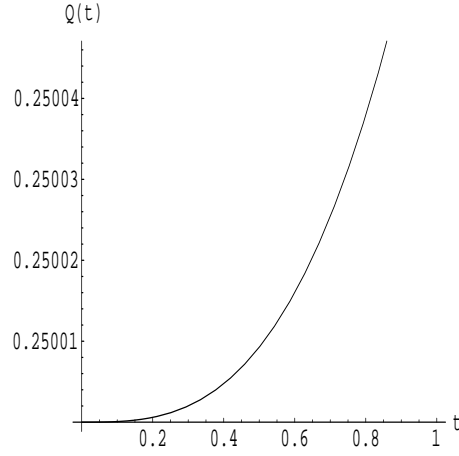


FIG. 10. Density radius of 1D oscillator in the random media as a function of time for short time scales, if initially it was in the ground state.

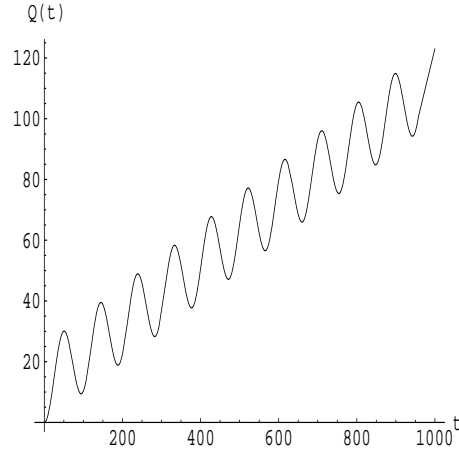


FIG. 11. Density radius of oscillator in the random media as a function of time for big time scales, if initially it was a small $a=0.1$ wave packet.

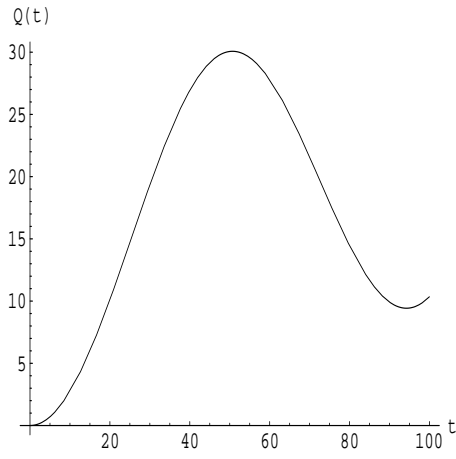


FIG. 12. Density radius of 1D oscillator in the random media as a function of time for average time scales, if initially it was a small $a=0.1$ wave packet.

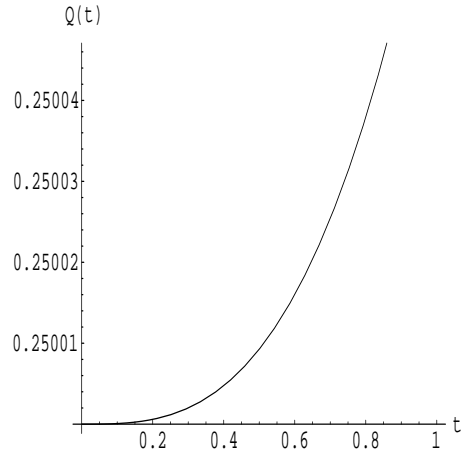


FIG. 13. Density radius of 1D oscillator in the random media as a function of time for short time scales, if initially it was a small $a=0.1$ wave packet.